STA 517 3.0 Programming and Statistical Computing with R

Generating Random Numbers Using the Inverse Transform Method

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1. Probability distribution functions in R to generate random numbers

rbeta	beta distribution	**rlnorm**	log-normal distribution
rbinom	binomial distribution	**rmultinom**	multinomial distribution
rcauchy	Cauchy distribution	**rnbinom**	negative binomial distribution
rchisq	chi-squared distribution	**rnorm**	normal distribution
rexp	exponential distribution	**rpois**	Poisson distribution
rf	F distribution	**rt**	Student's t distribution
rgamma	gamma distribution	**runif**	uniform distribution
rgeom	geometric distribution	**rweibull**	Weibull distribution
rhyper	hyper-geometric distribution		

There are other methods of generating random numbers from a particular distribution. In this lectorial we will discuss **Inverse Transform Method**.

2. Inverse transform method

Theorem 1: Probability Integral Transformation

> Let X have continuous cdf $F_X(x)$ and define the random variable Y as $Y = F_X(X)$. Then Y is uniformly distributed on (0, 1), that is, $P(Y \leq y) = y,\ 0 <$ y < 1.

Let's try to understand the theorem using an example.

Useful results to prove the theorem.

Result 1:

If F_X is strictly increasing, then F_X^{-1} is well defined by

$$F_X^{-1}(y) = x \Leftrightarrow F_X(x) = y.$$

If F_X is constant on some interval, then F_X^{-1} is not well defined by the above equation. To avoid this problem we define $F_X^{-1}(y)$ for $0 < y < 1$ by

$$F_X^{-1}(y) = inf\{x : F_X(x) \geq y\}.$$

Result 2:

If F_X is **strictly** increasing, then it is true that

$$F_X^{-1}(F_X(x)) = x.$$

Proof of Theorem 1:

For $Y = F_X(X)$ we have, for $0 < y < 1$, We can use Theorem 1 to generate random numbers from a particular distribution.

3. Steps in deriving random numbers using integral transformation method

1. Derive the cumulative distribution function of $f_X(x)$
2. Derive the inverse function $F_X^{-1}(u)$.
3. Write a function to generate random numbers.
    - Generate u from $Uniform(0, 1)$.
    - compute $x = F_X^{-1}(u)$.

Example 1

Write a function to generate n random numbers from the distribution with density $f_X(x) = 3x^2,\ 0 <$ x < 1.

Step 1: Find the cumulative distribution function of $f_X(x)$,

$$F_X(x) = x^3 \text{ for } 0 < x < 1$$

Step 2: Next we need to compute $F_X^{-1}(u)$,

$$F_X^{-1}(u) = u^{\frac{1}{3}}.$$

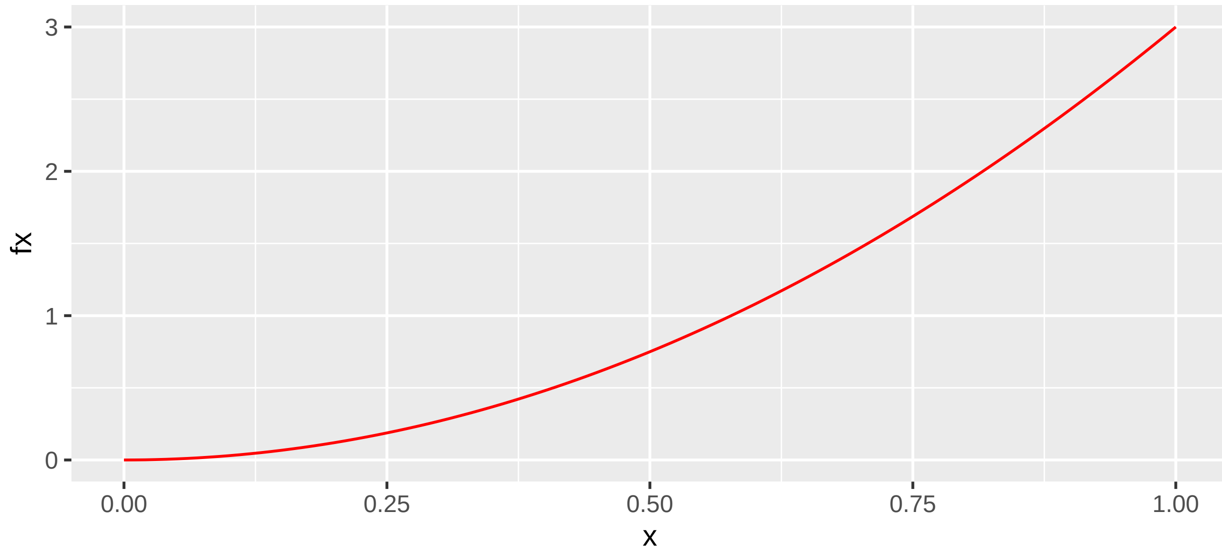
Step 3: R function

```
generate_it <- function(n){
  # Generate random numbers
  u <- runif(n)
  xgen <- u^(1/3)
  xgen

}

set.seed(2020)
generate_it(10)
```

```
 [1] 0.8648611 0.7332437 0.8520145 0.7812795 0.5143788 0.4069300 0.5054766
 [8] 0.7325562 0.1372012 0.8527963
```

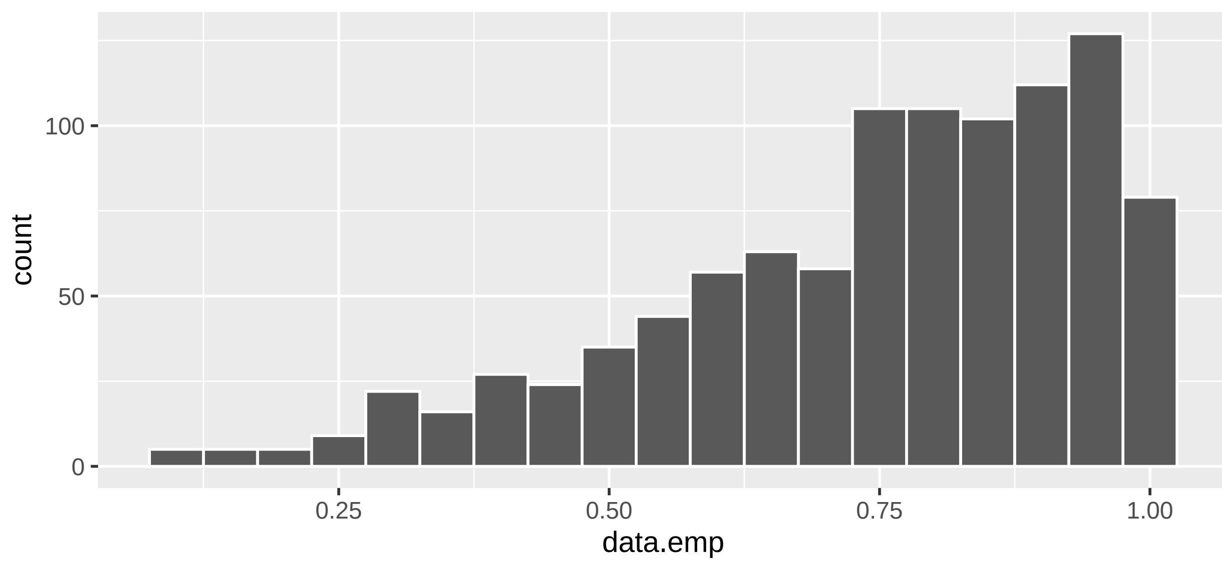
Visualisation of theoretical distribution

```r
library(tidyverse)
# Theoretical distribution values
theoretical.df <- tibble(x = seq(0, 1, 0.01), fx = 3*x^2)
ggplot(theoretical.df, aes(x = x, y = fx)) +
  geom_line(col = "red")
```



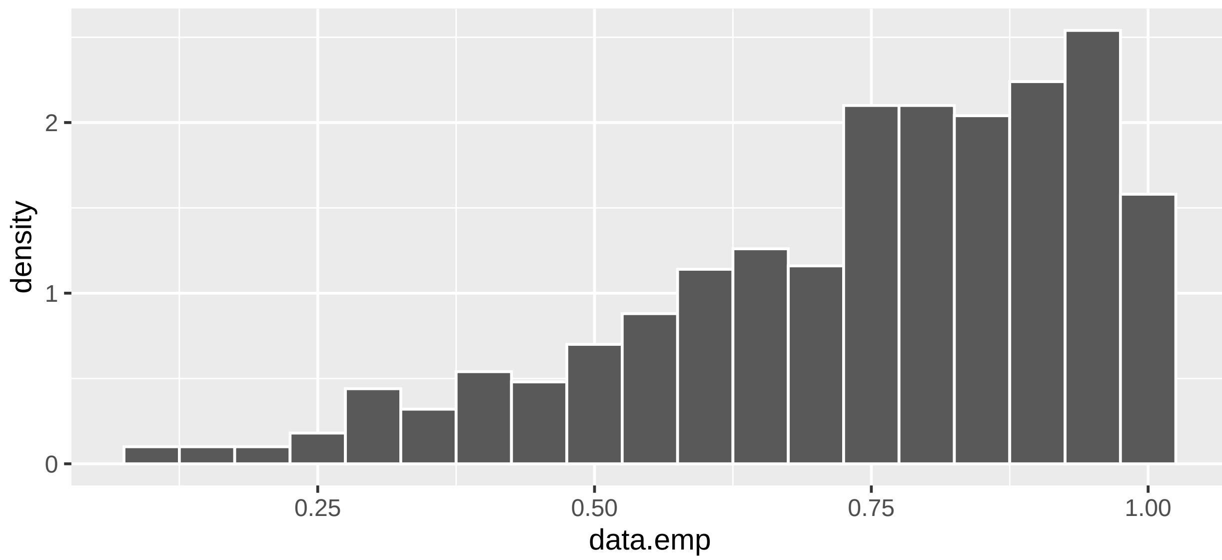
Visualize empirical distribution - counts

```r
empirical.df <- data.frame(data.emp = generate_it(1000))
# Plot empirical distribution - counts
ggplot(empirical.df, aes(x = data.emp))+
  geom_histogram(col = "white", binwidth = 0.05)
```



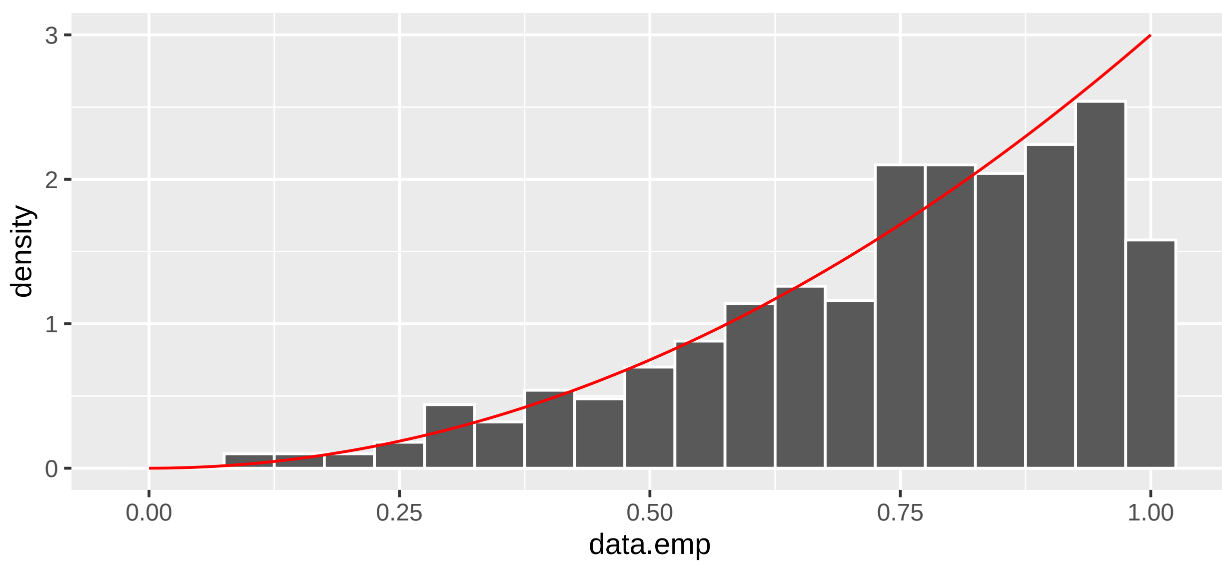
Visualize empirical distribution - density

```
ggplot(empirical.df, aes(x = data.emp, y=..density..)) +
  geom_histogram(col = "white", binwidth = 0.05)
```



Visualize theoretical distribution and empirical distribution together

```
ggplot(empirical.df, aes(x = data.emp, y=..density..)) +
  geom_histogram(col = "white", binwidth = 0.05) +
  geom_line(data = theoretical.df, aes(x = x, y = fx), color = 'red')
```



Function to generate random numbers and visualize theoretical and empirical distributions

```
generate_it_dist <- function(n){
  # Generate random numbers
  u <- runif(n)
  xgen <- u^(1/3)
  xgen

  # values for empirical distribution
  empirical.df <- data.frame(xgen=xgen)

  # values for the theoretical distribution
  theoretical.df <- tibble(x = seq(0, 1, 0.01),
  fx = 3*x^2)

  # arrange values and plot into a list
  list(
    xgen,
  ggplot2::ggplot(empirical.df, aes(x=xgen, y=..density..)) +
    geom_histogram(col="white", binwidth = 0.01) +
  geom_line(data = theoretical.df, aes(x = x, y = fx), color = 'red') )
}
```

Run the following codes and check the outputs.

```
# Sample size 10
generate_it_dist(10)

# Sample size 100
n100 <- generate_it_dist(100)
n100
n100[[1]]
n100[[2]]

# Sample size 10000
n10000 <- generate_it_dist(10000)
n10000[[2]]
```

Example 2

i) Write a function to generate random numbers from the $Exponential(\lambda)$ distribution using the inverse transformation method.

```
exp_generator <- function(n, lambda){
  u <- runif(n, 0, 1)
  exp.values <- -log(1-u)/lambda
    exp.values
}
```

ii) Generate 1000 random numbers from the $Exponential(2)$ distribution.

```
set.seed(111)
exp2 <- exp_generator(1000, 2)
```

```
exp2
```

```
   [1] 0.4494480553 0.6481923170 0.2313527655 0.3617246745 0.2371369396
   [6] 0.2709322984 0.0053575233 0.3799590208 0.2829583378 0.0491822557
  [11] 0.4057175747 0.4460777823 0.0347506843 0.0243577065 0.0849213804
  [16] 0.2956815133 0.0940352373 1.6986170219 0.1860149942 0.4765634595
  [21] 0.2819908410 0.1681050995 0.2093901924 0.2443915342 1.7136805265
  [26] 0.1943237064 0.5295459853 0.1665514095 0.7742370722 0.4530719863
  [31] 0.0301916961 0.3565727707 0.3134854035 0.3168348730 0.2229446628
  [36] 0.6248519440 0.0618275799 0.7662213541 0.5138077080 0.8176366795
  [41] 0.5124920900 0.1991144107 0.5048763728 1.3191629041 0.4281180826
  [46] 0.2283927700 0.2868891197 0.9687000140 0.4944247630 0.7893952961
  [51] 0.6457918773 0.4397922629 0.0166603330 0.2024786288 2.8594472510
  [56] 0.3973390414 0.4288139445 0.3046928120 0.0507830647 0.8187648467
  [61] 0.0004628645 0.3143768792 0.0951330393 0.1500275280 1.2583973452
  [66] 0.1319369315 0.0269988274 0.1814849470 0.0058975523 0.1788432265
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  [76] 0.2918641253 0.1555411929 2.0605831023 0.0499787653 0.7708174958
  [81] 0.0638327052 1.0652220376 0.0702625052 0.2556944548 0.0452983603
  [86] 0.2348416796 0.5772517799 0.6635839965 0.7369025382 0.4337328436
  [91] 0.3577976163 0.9586059319 0.4968845142 0.4325682393 0.6740164567
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```

```
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```

```
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[826] 0.0701276547 0.1240882941 0.3245141703 0.2639112699 0.3594617648
[831] 0.9104509200 0.3966305341 0.4056783645 2.3767650110 1.0723368649
[836] 0.1017004770 0.0295534861 0.7224027014 0.0012667110 0.2859878343
[841] 0.2747146952 0.3437436047 0.4542905453 0.2679356757 1.0830436528
[846] 0.0710358143 0.7927198037 0.2130347294 0.1534539459 1.0016691918
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[856] 0.0403738065 0.0282663022 0.9246619855 0.9224301199 0.2673285622
[861] 0.4230395972 0.3863112656 1.3191898048 0.1827931173 0.1337903558
[866] 0.2399728140 0.5773537356 0.7684652716 0.9656549280 0.0086901110
[871] 0.4929685277 0.5115543526 0.3387641721 0.3243455675 0.0003366354
[876] 0.0873460534 0.3294451876 1.9813291253 0.4855167325 0.2194847088
[881] 0.0309405037 0.5108227110 1.0726075611 0.3609839423 0.3820540126
[886] 0.0313118433 0.6440729947 0.4936194847 0.3337511917 1.9182473393
[891] 0.3546399074 0.0558454057 0.1334837326 0.3272070487 0.0554143874
[896] 0.2619027786 0.7494482948 0.0435722317 0.1592125303 0.9956577034
[901] 0.0994810928 0.0209276025 0.3935775150 0.3088993330 0.0045931547
[906] 1.8014414468 0.0300223617 0.7485592023 1.1044315033 0.1872001220
[911] 0.7502565939 0.1292379213 0.1121858748 0.6480451755 1.2422944493
[916] 0.5263227825 0.0400438936 0.9509638634 1.4212956820 0.1315167581
[921] 0.0634819213 0.4358357494 0.1167388588 0.8162951581 0.1827574226
[926] 0.1884679064 0.2300597011 0.1915938577 0.1775284251 0.2604890123
[931] 0.8904090588 1.4111424328 0.0392753031 0.7719081246 0.5394257817
[936] 0.0204700094 0.0841252882 0.2588852408 0.2424310396 0.6588303785
[941] 0.6704474212 0.2601026891 0.0691107828 1.3109838368 0.3330782044
[946] 0.7885720662 0.1058949850 0.8321418651 0.7607748470 1.7737045349
[951] 0.4465230859 0.1716820444 0.5606338791 0.5152859792 0.2807607845
[956] 0.1737200527 0.6385847419 0.3569037022 0.1284534322 0.2673521512
[961] 0.5457178427 0.0827301559 0.0955156410 0.0522736159 0.0037987583
[966] 0.0581840906 0.4248020255 0.6932740339 1.8484423218 0.3279396084
[971] 0.1358753976 0.5404927012 0.0688954097 0.0426506986 0.4796767093
[976] 0.0158038913 0.0726931498 0.2047496195 0.0402370098 0.1343942374
```

```
[981] 0.8972664487 0.1523066821 0.3331043420 0.0349435730 0.3183395265
[986] 0.2613156916 0.0185045379 0.1067139110 0.8352169333 0.7749512387
[991] 1.3703322469 0.2839747783 0.5716211491 0.6238504269 0.6111759897
[996] 2.3721048818 0.2031257786 0.7555383871 0.2252286640 0.4334517398
```

iii) Graph the density histogram of the sample with the $Exponential(2)$ density superimposed for comparison.

```
# values for empirical distribution
empirical.df.exp <- data.frame(xgen=exp2)
# values for the theoretical distribution
theoretical.df <- tibble(x = seq(0.1, 5, 0.01),
  fx = 2*exp(-2*x))

ggplot(empirical.df.exp, aes(x = xgen, y=..density..)) +
  geom_histogram(col = "white", binwidth = 0.05) +
  geom_line(data = theoretical.df, aes(x = x, y = fx), color = "forestgreen") +
  labs(x="x", y="fx")
```